WEIGHTED LEAST-SQUARES VERSUS MAXIMUM LIKELIHOOD ESTIMATES OF NON-HIERARCHICAL MODELS

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In this paper we analyze two sets of categorical data with the following objectives: (1) to show that fitting non-hierarchical models is very feasible and that such models may give extremely good fit, when hierarchical models may not, and (2) to assess the relative utility of maximum likelihood (ML) and weighted least-squares (WLS) estimation techniques in light of data structure and available computer programs. We assume familiarity with the basic hierarchical log-linear technique as presented, for example, in Goodman [5, 6, 7].

Illustration 1: Self-esteem data

The data in Table 1, taken from Rosenberg [11], show proportions of persons with "high" self-esteem by religion (three categories) and father's education (six categories). We take self-esteem to be a dichotomous variable ("high" or not "high") and treat it as the dependent variable. When using the ML hierarchical method [6] in this data situation, one typically starts by testing whether father's education and religion interact in their effects on self-esteem, and, if that three-way interaction is absent, the next step would be to inquire whether the main effects of father's education and religion are significant. The first step, then, is to fit the three two-way marginals, which is equivalent to "fitting" the hypothesis of no three-way interaction [3]. In this case, such a model fits the data very poorly ($\chi^2 = 37.66$, 10 df, $p < .001$), and the investigator would infer that religion and father's education do indeed interact in their effects on self-esteem. At this point the typical log-linear analysis would stop, for the only model among the hierarchical ones that would give a better fit would be the so-called "saturated model," which, because it uses all the degrees of freedom available, yields no data reduction whatsoever, enabling the analyst to do no more than describe the observed frequencies in the table.

We shall now briefly describe how one may try to identify a parsimonious non-hierarchical model that fits the data extremely well. We first fit the saturated model, using a program called NONMET which is a WLS routine [8]. (For documentation, write to the Institute for Research in Social Science, University of North Carolina, Chapel Hill.) In order to fit any model using the NONMET program, we must first specify the design matrix using the so-called "effect coding" [10, pp. 121-128]. Given below are the first eight columns of the design matrix. These columns represent the "general mean" and the "main effects" of religion (two components, R_1 and R_2) and father's education (five components, E_1, ..., E_5). The ten remaining columns in the design matrix (not shown) correspond to the interaction effects. They can be derived by multiplying corresponding elements of one column (R_1 or R_2) for religion and one (E_1, E_2, E_3,

Row	Columns of Design Matrix							
	(1) $\bar{X}$	(2) R_1	(3) R_2	(4) E_1	(5) E_2	(6) E_3	(7) E_4	(8) E_5
C:1	1	1	0	1	0	0	0	0
2	1	1	0	0	1	0	0	0
3	1	1	0	0	0	1	0	0
4	1	1	0	0	0	0	1	0
5	1	1	0	0	0	0	0	1
6	1	1	0	-1	-1	-1	-1	-1
J:1	1	0	1	1	0	0	0	0
2	1	0	1	0	1	0	0	0
3	1	0	1	0	0	1	0	0
4	1	0	1	0	0	0	1	0
5	1	0	1	0	0	0	0	1
6	1	0	1	-1	-1	-1	-1	-1
P:1	1	-1	-1	1	0	0	0	0
2	1	-1	-1	0	1	0	0	0
3	1	-1	-1	0	0	1	0	0
4	1	-1	-1	0	0	0	1	0
5	1	-1	-1	0	0	0	0	1
6	1	-1	-1	-1	-1	-1	-1	-1

E_4, or E_5) for father's education. This 18 by 18 design matrix when used in NONMET yields estimates of the parameters of the saturated model.

Note that the WLS routine programmed in NONMET predicts the proportion of persons with "high" self-esteem or the logarithm or the logit thereof. Here we confine ourselves to the logit form. (Logit is defined as the natural logarithm of $p/(1-p)$, where p is the proportion of cases with "high" self-esteem.)

With the ML model we are not predicting the logit of the dependent proportion, but rather the logarithm of the frequency, or the logarithm of the proportion of the total number of cases in each cell of the three-way table of religion-by-father's education-by-self-esteem [1, 5, 6]. (Let us call the ML model the log-frequency model.) Since, with this model we predict the cell frequencies rather than the logits, we are now predicting 36 values. Therefore, the design matrix needed to predict the observed frequencies in the three-way table will have 36 rows, one corresponding to each cell in the table. The matrix for the saturated log-frequency model will also have a total of 36 columns, for we must now estimate parameters representing the general mean (1), main effects (self-esteem, 1; religion, 2; father's education, 5), the two-way effects (self-esteem and religion, 2; self-esteem and father's education, 5; religion and father's education, 10) and the three-way effects (10). It is easy to demonstrate that the parameters involving self-esteem for the saturated log-frequency model are exactly half of the corresponding parameters of the logit model [6].

For the saturated model, the ML estimates can be easily obtained using a program such as ECTA. We can also use a more general ML estimation program, such as MAXLIK [9], and such a program must be used to obtain estimates for any model other than the hierarchical variety described by Goodman.

The WLS estimates and their standard errors for the saturated logit model described above are shown below:

R_1:	-.1108(.0636)	R_1E_3:	.3418(.1016)
R_2:	.3334(.0817)	R_1E_4:	-.0132(.1474)
E_1:	-.1900(.1200)	R_1E_5:	-.3531(.1556)
E_2:	-.0907(.0841)	R_2E_1:	-.1655(.2164)
E_3:	-.2569(.0815)	R_2E_2:	-.3214(.1609)
E_4:	.0592(.1115)	R_2E_3:	-.0366(.1395)
E_5:	.2403(.1169)	R_2E_4:	-.0348(.1799)
R_1E_1:	.1007(.1375)	R_2E_5:	-.4508(.1975)
R_1E_2:	.0202(.1042)		

(It can be easily shown that for the saturated model the WLS estimates and the ML estimates are identical.)

Each estimate shown above can be used to test the significance of the corresponding parameter by calculating the statistic (estimate/standard error)^2, which is distributed asymptotically as chi-square with one degree of freedom. This procedure yields the following parameters as "significant": R_1, R_2, E_1, E_3, E_5, R_1E_3, R_1E_5, R_2E_2, and R_2E_5. Fitting a model containing only these parameters and the grand mean may provide a parsimonious representation of the data in question. Such a model would leave 8 degrees of freedom, since it fits only 10 of the 18 parameters of the saturated model. However, in this case we can further reduce the number of parameters that need to be estimated by fitting (E_1 - E_5) instead of the pair E_1 and E_5, and $(R_1-R_2)E_5$ instead of the pair R_1E_5 and R_2E_5. With this further reduction, we estimate a total of 8 parameters, leaving 10 degrees of freedom. The design matrix for this last model is shown below:

Row	Columns of Design Matrix							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	X	R_1	R_2	E_1-E_5	E_3	$(R_1-R_2)E_5$	R_1E_3	R_2E_2
C:1	1	1	0	1	0	0	0	0
2	1	1	0	0	0	0	0	0
3	1	1	0	0	1	0	1	0
4	1	1	0	0	0	0	0	0
5	1	1	0	-1	0	1	0	0
6	1	1	0	0	-1	-1	-1	0
J:1	1	0	1	1	0	0	0	0
2	1	0	1	0	0	0	0	1
3	1	0	1	0	1	0	0	0
4	1	0	1	0	0	0	0	0
5	1	0	1	-1	0	-1	0	0
6	1	0	1	0	-1	1	0	-1
P:1	1	-1	-1	1	0	0	0	0
2	1	-1	-1	0	0	0	0	-1
3	1	-1	-1	0	1	0	-1	0
4	1	-1	-1	0	0	0	0	0
5	1	-1	-1	-1	0	0	0	0
6	1	-1	-1	0	-1	0	1	1

The WLS estimates of the parameters of this model and their standard errors are:

R_1:	-.1112(.0564)	$(R_1-R_2)E_5$:	-.3052(.1325)
R_2:	.3344(.0743)	R_1E_3:	.3606(.0709)
E_1-E_5:	-.1435(.0692)	R_2E_2:	-.2839(.0933)
E_3:	.2565(.0649)		

The corresponding ML estimates obtained by using the MAXLIK program are:

R_1:	-.1124	$(R_1-R_2)E_5$:	-.3096
R_2:	.3370	R_1E_3:	.3604
E_1-E_5:	-.1444		
E_3:	-.2572	R_2E_2:	-.2846

The goodness of fit chi-square for the WLS procedure is 3.15 (10df, p. = .978) and that for the ML procedure is 3.18 (10df, p = .977).

Given the near identity of the estimates obtained using the two techniques, the analyst would be advised to employ the technique that is easiest to use, and less expensive. Since the ML programs such as MAXLIK are quite costly and require the input of initial estimates, which must be obtained from prior calculations, it seems clear that for data where we can specify one of the variables as dependent and where that variable is dichotomous, it is preferable to use the WLS technique. It deserves to be emphasized that the analysis procedure just described makes sense only if all the parameters can be given substantive interpretations. (The theoretical significance of these particular parameters, of course, will have to be found on the basis of nonstatistical considerations that are beyond the scope of this paper.) We wish merely to illustrate that these eight parameters suffice to describe virtually all of the variation in self-esteem in the given data set.

Illustration 2: Openness to Change

The second illustration uses data in which the dependent variable is a polychotomy. The data are from a survey reported in Duncan [2].

In that survey, respondents were asked about their attitudes toward "making changes in the way our country is run" [2, pp. 177-181]. Four response categories were used: R_1: "We should rarely, if ever, make changes", R_2: "We should be very cautious in making changes", R_3: "We should feel free to make changes", R_4: "We must constantly make changes." Suppose we are interested in assessing the effect of the year (Y) of the survey (1956 or 1971) and the respondent's political party identification (P_1: Republican, P_2: Democrat, or P_3: Independent).

Analyzing these data using the WLS technique to "search" for a parsimonious model is not as straightforward as in the previous illustration, because the dependent variable is now polychotomous. We start by viewing the response-by-party-by-year distribution as a multinomial (with 24 classes) and let the multinomial proportion in each cell take the place of the binomial proportion in Illustration 1, treating log p as the quantity that is predicted. It is important to note that the procedure is statistically incorrect and may produce bias in our estimates of the variance-covariance matrix. Nevertheless, as we demonstrate below, this procedure is less problematic than it might appear at first blush. (We employ the WLS procedure only for a preliminary analysis, which we follow up with the ML procedure. In the present case the ML estimates turn out to be substantially similar to the WLS estimates.)

Our strategy involves first fitting the saturated model to the data in Table 2. By omitting non-significant effects, specifying "difference" effects for pairs of related effects, and taking other similar steps, we obtain, after a few preliminary runs using NONMET, a parsimonious model that seems to fit the data extremely well. This model is then fitted using the MAXLIK program. The results are reported in Table 3.

In the analysis of these same data, Duncan [2] uses standard hierarchical ML procedures to arrive at a parsimonious and good-fitting model, by treating each polychotomous variable with _g_ categories as a set of _g_ dichotomous variables, in each case contrasting a given category with the _g-1_ remaining categories. He terms these dichotomous variables "formal" variables.

Duncan begins the analysis of these data by fitting a baseline model which posits independence between response and the joint variable year-by-party. He then proceeds to fit several models one at a time, each of which includes the parameters of the baseline model as well as one additional parameter representing the effect on one of the response categories of one of the independent variable categories. Thus, he shows twelve effects for party and response and four effects for year and response. The statistical significance of each additional parameter is ascertained by comparing the difference in chi-square of each model and the baseline model. Each of the twelve three-way interactions are also tested for significance one at a time, by comparing a model that included it with a model that excluded it. In this way, Duncan arrives at what we will for convenience term his "best-fitting" model, i.e., his Model (15), with 15 parameters, nine degrees of freedom, and a chi-square value of 3.5, with a probability exceeding .9.

Duncan's "best-fitting" model can be specified in a design matrix format: First think of each row of the matrix as being defined by the eight dichotomous formal variables, rather than by the underlying three variables. Then, for each of the formal variable effects, one can specify a column with a 1 for each cell corresponding to the category of the formal variable, and a -1 for all other cells. These are the so-called elementary column specifications shown in columns (1) through (8) in Table 4. These elementary columns can in turn be used to specify any of the joint (interaction) effects in the model. Thus, for example, column (9) shows the vector corresponding to the parameter R_4Y, which is obtained by multiplying corresponding elements in elementary columns (4) and (8). Columns (10) and (11) of Table 4 show two other interaction vectors that are specified according to the formal variable approach. _The formal approach is used only to define the interaction terms in the model_--terms involving the multiplication of two or more elementary vectors. It is important to note that _all_ formal variable interaction terms involving _a_ given polychotomous variable cannot be simultaneously included in the design matrix, because any one column for the _g_-category polychotomous variable would then be linearly dependent on the other _g-1_ columns. Likewise, we cannot use the vectors in the first eight columns of Table 4 to specify the main effects. Because of this, we resort to the more usual column specification for fitting main effects and specify all interaction effects in terms of formal variables in the manner described above and shown in Table 4.

Model (1) in Table 5 corresponds to Duncan's best-fitting model. We arrive at the design matrix specification for this model by specifying baseline model parameters according to the usual column design, with the additional interaction effects specified according to the formal variable design. Note that this design matrix specification yields results identical to Duncan's best-fitting model, and it allows for estimation of parameters without resorting to procedures for handling structural zeros.

The second model in Table 5 is a non-hierarchical model fitted to the same data (Table 2). Once a matrix approach is adopted, there is no need for the investigator to adhere only to hierarchical models. Note that the terms in model (2) are quite similar to those in model (1). Model (2) differs from model (1) in that the former excludes the effects P_1 and P_2Y and includes the effects R_1P_2Y and R_3P_2Y. This substitution of two parameters leaves unchanged the degrees of freedom but improves the fit of the model. In addition, the substantive inferences made on the basis of model (2) would differ from those made with model (1), corresponding to the two different parameters in the models.

It is interesting to note that not only do models (1) and (2) in Table 5 allow us to make slightly different inferences about the relationships in the data in Table 2, but also that both of these models differ substantially in structure from the parsimonious and slightly better fitting model in Table 3. Neither the different parameterization of the design matrix in our approach nor the introduction of the formal variable format accounts for any of the differences in structure between the model in Table 3 and the models in Table 5, since if the design matrix for the model in Table 3 were recast in terms of the formal variables there would be no change in the parameter estimates or goodness-of-fit statistic. We may remind the reader of Goodman's caution [4, p. 48] that different selection procedures may lead to different models, all of which concisely fit a given set of data. Another analyst may obtain yet another model that fits these data well.

Nevertheless, we wish to underscore the central point that any analyst is well-advised to consider non-hierarchical models as well as those that are hierarchical. Duncan's analysis of these data was restricted to hierarchical models, and was somewhat cumbersome in requiring that many models be fitted in order to arrive at one that was reasonably parsimonious.

Our initial analysis of the data in Table 2 yielded in straightforward fashion a very parsimonious model that was non-hierarchical. Reanalysis of the data using the formal variable approach with a design matrix allowed us to duplicate Duncan's finding, and then go on to fit a non-hierarchical model that describes the data somewhat better than does Duncan's model.

General Recommendations

(1) If the dependent variable is dichotomous, use a WLS program such as NONMET, rather than a hierarchical ML program such as ECTA. This allows the analyst to use his or her imagination in finding models that are theoretically appropriate, parsimonious, and that fit the data very well. Although such models may occasionally be hierarchical, in which case ECTA may be useful, a WLS program such as NONMET can be used to fit all the models fitted by ECTA and more. A program such as MAXLIK will then allow the analyst to obtain ML estimates, once the appropriate model has been found with the WLS technique.

(2) If the dependent variable is polychotomous, first apply a WLS program (e.g., NONMET) to the logarithm of cell frequencies to find the most suitable model as in the second illustration. Since the NONMET program in this case is statistically less attractive because it employs the wrong variance-covariance matrix, use a program such as MAXLIK to obtain ML estimates for the model.

(3) For a logit model, in which the dependent variable is dichotomous, it would be enough to use the NONMET program alone, without finding ML estimates, because both procedures yield virtually identical results.

(4) For a set of data with a polychotomous dependent variable, it may be the case that the use of the NONMET program in the manner we suggest above, although statistically not quite attractive, produces acceptable results for most analyses. For our example, the WLS estimate did not depart significantly from the ML estimates.

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TABLE 1

PROPORTION WITH HIGH SELF ESTEEM, BY RELIGIOUS BACKGROUND AND FATHER'S EDUCATION

Religion*	Father's Education					
	Eighth grade or less	Some high schl.	H.S. grad.	Some col.	Col. grad.	Post-grad.
C:	.681	.685	.717	.709	.675	.729
(n)	(360)	(482)	(541)	(141)	(114)	(70)
J:	.718	.706	.745	.788	.879	.827
(n)	(39)	(126)	(137)	(85)	(99)	(75)
P:	.648	.720	.525	.699	.706	.738
(n)	(193)	(325)	(406)	(156)	(279)	(122)

*C: Catholic; J: Jewish; P: Protestant

TABLE 2

PERCENTAGE DISTRIBUTIONS OF RESPONSES TO "CHANGE" QUESTION, BY YEAR AND PARTY

Year & Response	Party: Rep. (P_1)	Dem. (P_2)	Ind. (P_3)
1956 (Y_1)			
R_1	2.0	2.1	1.1
R_2	47.0	49.0	51.1
R_3	37.0	38.1	31.1
R_4	14.0	10.9	16.7
Total	100.0	100.0	100.0
(n)	(200)	(431)	(90)
1971 (Y_2)			
R_1	1.3	2.2	0.4
R_2	53.4	42.4	36.0
R_3	25.8	32.6	33.0
R_4	19.5	22.8	30.6
Total	100.0	100.0	100.0
(n)	(159)	(509)	(242)

TABLE 3

ESTIMATES FOR PARSIMONIOUS MODEL FITTED TO DATA IN TABLE 6

Parameter	WLS Estimate	ML Estimate
R_1	-2.2824	-2.2958
R_2	1.1862	1.1910
R_3	.8513	.8540
P_1	- .2710	- .2728
P_2	.7779	.7816
Y	.0960	.0961
P_1Y	- .3009	- .3022
P_2Y	.0521	.0510
R_1P_2	.2944	.3022
R_1Y	- .3220	- .3196
R_1P_2Y	.2901	.2837
$R_2(P_1-P_2)$	.0847	.0875
$R_2(P_1-P_2)Y$	.1326	.1315
R_3P_2Y	- .1529	- .1519
Chi-square		
value	2.5720	1.6214
df	9	9
p	.9789	.9774

TABLE 4

COLUMN SPECIFICATION FOR DESIGN MATRIX CORRESPONDING TO FORMAL VARIABLE APPROACH

								Elementary Column Specifications								Interaction Columns		
								(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
R_1	R_2	R_3	R_4	P_1	P_2	P_3	Y	R_1	R_2	R_3	R_4	P_1	P_2	P_3	Y	R_4Y	R_1P_3	R_2P_1Y
1	2	2	2	1	2	2	1	1	-1	-1	-1	1	-1	-1	1	-1	-1	-1
1	2	2	2	1	2	2	2	1	-1	-1	-1	1	-1	-1	-1	1	-1	1
1	2	2	2	2	1	2	1	1	-1	-1	-1	-1	1	-1	1	-1	-1	1
1	2	2	2	2	1	2	2	1	-1	-1	-1	-1	1	-1	-1	1	-1	-1
1	2	2	2	2	2	1	1	1	-1	-1	-1	-1	-1	1	1	-1	1	1
1	2	2	2	2	2	1	2	1	-1	-1	-1	-1	-1	1	-1	1	1	-1
2	1	2	2	1	2	2	1	-1	1	-1	-1	1	-1	-1	1	-1	1	1
2	1	2	2	1	2	2	2	-1	1	-1	-1	1	-1	-1	-1	1	1	-1
2	1	2	2	2	1	2	1	-1	1	-1	-1	-1	1	-1	1	-1	1	-1
2	1	2	2	2	1	2	2	-1	1	-1	-1	-1	1	-1	-1	1	1	1
2	1	2	2	2	2	1	1	-1	1	-1	-1	-1	-1	1	1	-1	-1	-1
2	1	2	2	2	2	1	2	-1	1	-1	-1	-1	-1	1	-1	1	-1	1
2	2	1	2	1	2	2	1	-1	-1	1	-1	1	-1	-1	1	-1	1	-1
2	2	1	2	1	2	2	2	-1	-1	1	-1	1	-1	-1	-1	1	1	1
2	2	1	2	2	1	2	1	-1	-1	1	-1	-1	1	-1	1	-1	1	1
2	2	1	2	2	1	2	2	-1	-1	1	-1	-1	1	-1	-1	1	1	-1
2	2	1	2	2	2	1	1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1
2	2	1	2	2	2	1	2	-1	-1	1	-1	-1	-1	1	-1	1	-1	-1
2	2	2	1	1	2	2	1	-1	-1	-1	1	1	-1	-1	1	1	1	-1
2	2	2	1	1	2	2	2	-1	-1	-1	1	1	-1	-1	-1	-1	1	1
2	2	2	1	2	1	2	1	-1	-1	-1	1	-1	1	-1	1	1	1	1
2	2	2	1	2	1	2	2	-1	-1	-1	1	-1	1	-1	-1	-1	1	-1
2	2	2	1	2	2	1	1	-1	-1	-1	1	-1	-1	1	1	1	-1	1
2	2	2	1	2	2	1	2	-1	-1	-1	1	-1	-1	1	-1	-1	-1	-1

TABLE 5

ML ESTIMATES OF PARAMETERS AND GOODNESS OF FIT CHI SQUARES OF TWO PARSIMONIOUS MODELS FOR DATA IN TABLE 2: MODELS SPECIFIED USING FORMAL VARIABLE APPROACH

Parameter	Model 1	Model 2
R_1	-2.4839	-2.7211
R_2	1.2535	1.3323
R_3	.8991	.9763
P_1	- .1159	--- a
P_2	.8443	.9437
Y	.2729	.2534
P_1Y	- .2560	- .2873
P_2Y	- .0654	--- a
R_2Y	.0501	.0529
R_4Y	.1909	.1862
R_1P_3	- .2706	- .4309
R_2P_1	.0471	.0443
R_4P_3	.0929	.0960
R_1P_2Y	--- a	.0595
R_2P_1Y	.0737	.0891
R_3P_2Y	--- a	.0350
Chi-square		
value	3.477	2.643
df	9	9
p	.942	.977

a Parameter excluded from the model